VOLUME XXII NUMBER 4

THE

JOURNAL OF GEOLOGY

MAY-JUNE 1914

THE STRENGTH OF THE EARTH'S CRUST

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PART IV. HETEROGENEITY AND RIGIDITY OF THE CRUST AS MEASURED BY DEPARTURES FROM ISOSTASY

INTRODUCTION AND SUMMARY

In Part I were examined certain geologic tests of the strength of the crust; in Part II the geodetic evidence in regard to the effective areal limits of rigidity; in Part III the influence of variable vertical distribution of density. All three lines of investigation converge toward showing the rigidity of the outer crust—the zone of isostatic compensation—to be such that very considerable stresses can be carried over areas whose radii range between 100 and 300 km. There arise to be considered next the following problems: first, the variability in depth of compensation and its influence; second,

whether the stresses represented by the incompleteness of isostasy are carried by the rigidity of the outer crust, or are transferred in some measure to the deeper body of the earth; third, the magnitudes of the stresses, measured in terms of loads, which are indicated by the gravity anomalies and deflection residuals.

It is found in answer that under the hypothesis which forms the basis of Hayford's work, that of uniform compensation, complete at a given depth, there are indications, given by comparing different areas, of a great range in the depth of the bottom. Under an assumption which is probably nearer to nature—that is, the hypothesis of a variable and gradually disappearing compensation—there is room for even a greater heterogeneity of the crust and a greater variability in the depth reached by the zone of compensation. But, on the other hand, it is concluded that the zone of compensation, as an outer rigid crust separated from the rigid inner earth by an intervening zone of lowered rigidity, is a reality in earth structure. The stresses due to the heterogeneities of density and relief within and upon this crust appear to be borne by the crust, not by the inner earth. Under the third subject it appears, upon review of the evidence given by the deflection residuals, that these may be interpreted so as to show departures from equilibrium comparable to the results given by the gravity anomalies, instead of the 250 feet which Hayford thought to exist. The two independent lines of geodetic investigation are thus seen to agree and it may be concluded with some confidence that the individual isostatic regions of the United States are on the average between 600 and 900 feet out of equilibrium. Evidence from other parts of the world appears to show, furthermore, that a number of regions exhibit greater departures from isostasy than those observed within the United States. The strain imposed on the crust by the Niger Delta, though large, is apparently not as large as some made known by geodetic measurements.

Thus from various directions of attack the crust is shown to be an earth shell of high rigidity and consequently high elasticity. Geodetic evidence justifies the view, brought forward by geologic evidence, that the delta of the Niger is to be looked upon as supported by the strength of the crust.

VARIABLE OR CONSTANT DEPTH OF COMPENSATION

The Cordilleran region awoke to an era of great orogenic and igneous activity near the beginning of the Tertiary, and, especially in the Neocene, has become broadly elevated into one of the great plateau regions of the world. Large areas like the Colorado plateaus, which since the beginning of the Paleozoic had rested near sea-level, at times beneath and again slightly above, have been lifted many thousands of feet. Block-faulted structures indicate the dominance of vertical forces rather than surficial compression as the cause of these movements. The uplift has not been of the nature of a broad even upwarp, and adjacent regions show great contrasts in elevation. These different surface results of the interior forces suggest differences in elevatory forces at comparatively shallow depths. The region is known to be in a fair degree of isostatic equilibrium notwithstanding the high relief. Davis has shown why these movements cannot be regarded as differential sinkings toward the center of the earth.[1] These features suggest, then, subcrustal decreases in density during the Tertiary as a cause of the broad movements of elevation.

The rising of great bodies of magma to high levels in the zone of isostatic compensation, their irregular distribution, the great quantities of heat and gases which would invade the roofs are suggested by the observed evidences of regional igneous activity at the surface as the probable causes of the changes in density and regional vertical movements. A consequence of such a cause would be a lessened strength of the crust to resist strain, a lessened depth to the zone of isostatic compensation, and a decreased size of the unit areas departing from equilibrium.

The history of the Cenozoic in the Cordillera has repeated the history of other regions at other times, either in the Archean igneous activity or later. The slow conduction of this excess heat from the outer crust, the solidification of the reservoirs of magma, would, in the course of ages, bring about a new rigidity. Upon disturbances of the equilibrium by erosion or compressive forces there would be found a new and greater depth to the zone of compensation.

[1] "Bearing of Physiography upon Suess's Theories Abstract," *Intern. Geog. Cong., 8th Report*, 1905, p. 164; *Amer. Jour. Science* (4), XIX (1905), 265–73.

Where ages of uplift and erosion have followed periods of igneous activity there are revealed great bodies of intrusive rock varying in density from granites at 2.65 to gabbros at 3.0. These great batholiths are of irregular distribution in the crust, both vertically and horizontally. Their abundance increases downward so far as erosion has revealed the evidence. The outer crust of the earth has become vertically and areally heterogeneous by such means and should cause variations and irregularities to an appreciable degree in the distribution of isostatic compensation, as noted under the topic of the influence of variable rate of compensation upon gravity anomalies. Here we note in addition the decreased depth of compensation and decreased rigidity at the time of intrusion.

Hayford notes that the stations classified into geographic groups show as a rule as great contradictions in depths of compensation between adjacent groups as in those which are far apart. This variation between adjacent groups is taken by him as weakening the evidence that there is any real variation in the depth of compensation over the whole area investigated.[1] For the reasons outlined previously, the present writer, influenced by the geologic inferences, does not view such irregularity of distribution as proof that the evidence is weak and conflicting. The strength of the evidence must be judged rather by the nature of the residuals. Hayford points out that the depth of compensation in the West seems on the whole to be somewhat less than in other parts of the United States, though he does not regard it as safe to assert that it does exist. On dividing the whole area into four sections, the minimum sum of the squares of the residuals indicates depths as follows as best satisfying the hypothesis of uniform distribution of compensation:

From all residuals of the central group, 174 km.
From all residuals of the northeastern group, 187 km.
From all residuals of the southeastern group, indeterminate.
From all residuals of the western group, 107 km.[2]

In the 1909 paper Hayford gives a tabulation of the residuals for fourteen geographic groups. The results for the United States as a

[1] 1909, pp. 58, 59. [2] 1906, pp. 142–46.

whole and for the groups showing the shallowest, deepest, and the most irregular compensation are quoted below.[1] That solution is regarded as nearest the truth which gives the smallest mean value of the squares of the residuals.

TABLE XIX

PROBABLE DEPTHS OF COMPENSATION

Group	No. of Residuals δ	Mean Values of the Squares of the Residuals in Various Groups					Most Probable Depth
		Solution B Infinite Depth	Solution E 162.2 Km.	Solution H 120.9 Km.	Solution G 113.7 Km.	Solution A 0.0 Km.	Km.
United States (all observations)	733	146.50	14.05	**13.73**	13.75	25.77	**122.2**
Group 12 (parts of Minn., N.Dak., S.Dak., Neb., Kan.)	36	196.57	**7.00**	7.47	7.59	11.46	**305**
Group 5 (Mich., Minn., Wis.)	52	34.97	**23.60**	23.64	23.67	27.53	152
Group 8 (parts of Utah, Nev., Cal.)	42	128.97	22.27	18.79	**18.25**	35.78	**66**
United States (residuals multiplied by 1.327 to compare with Group 8)	....	194.40	18.65	**18.23**	**18.25**	34.21	....

It is seen that the mean value of the squares of the residuals in group 12 with most probable depth of 305 km. is considerably less than for the United States as a whole, in part no doubt owing to the moderate relief, yet the differences between the residuals in group 12 for the different solutions is much more pronounced than for the United States as a whole. The number of stations, 36, is large enough so that this can hardly be regarded as accidental. On the contrary, it would appear that for the whole United States the group differences are sufficient to mask in part the accuracy of the mean result of 122 km. and that the depth of compensation within certain groups is more reliable than for the United States as a whole.

In group 8 with most probable depth of 66 km. the mean value of the squares of the residuals is nearly 50 per cent higher than for

[1] 1909, pp. 55–58.

the United States as a whole, a value which may be ascribed to the mountainous relief and the support of individual mountains and ranges by the rigidity of the crust. Nevertheless the residuals for the several solutions fall into a somewhat regular system, and solutions E, H, and G are more sharply differentiated from the most probable one than for the whole United States. They may be compared better with the latter if the residuals for the whole country are multiplied by 1.327 as a factor in order to give the same numerical value under solution G. This is done at the bottom of the table. It would appear from these figures as though the arguments previously given from geologic analysis receive considerable support from the geodetic results and point to a much shallower depth for isostatic compensation in the Great Basin than over certain other portions of the United States. Furthermore, in the examination of the question of local versus regional compensation, it was only the forty mountain stations classified into two groups according to elevation which gave any suggestion that regional compensation to a radial distance of 166.7 km. was not about as probable as more local compensation. In these two lines of geodetic evidence as to limited depth and breadth of compensation there are suggestions therefore which support the geologic inference that the crust of the Cordilleran region may be weaker than over the United States as a whole. On the other hand, the warping or faulting-down of ancient continental areas into marginal sea-bottoms implies an increasing density of the subcrust and therefore possibly an increasing rigidity and strength under such areas. Such a contrast between the Atlantic Ocean bottom and the Great Basin would correspond to the great strength of crust necessary to sustain the delta of the Niger as compared with the moderate rigidity found by Gilbert for the crust beneath extinct Lake Bonneville, located within the limits of group 8.

The regions of shallower compensation in the United States are on the whole marked probably by a higher temperature gradient, the regions of deep compensation by a lower. This is illustrated by the very high gradient of the Comstock mine in Nevada and the very low gradient which is found in the Lake Superior copper mines. The temperature gradient may measure the depth to a zone of low

rigidity, determined by a certain relation of temperature and pressure.

Within an overlying zone of high rigidity, even where it is of uniform depth, the geodetic measurements of the depth of compensation may not, however, show uniformity. If the density is unequally distributed, the compensation of a region may be nearly completed at some depth above the base of the rigid zone, the lower part consisting of rock of mean density and therefore not possessing influence. A region of deep and marked rigidity, if characterized by notable irregularities in the distribution of either density or relief, would show large residuals. A region characterized by more uniform distribution of density and gentle relief would show lower residuals even with the same rigidity. A region with deep compensation would show within the limits of the group lower residuals for the same degree of uniform compensation, than where compensation was at lesser depths, since the attracting masses are spread over a greater distance.

As applications and tests of these principles, it is to be noted that group 5, embracing the Lake Superior region with its low-temperature gradients, has the highest residuals of any group in the United States. Further, the mean values of the least squares for the different solutions show less differentiation than in any other group. These facts suggest irregular distribution of density, high rigidity, and the zone of rigidity may extend below the most probable depth, 152 km., indicated for the limits of compensation. The topographic deflections are only 58 per cent compensated. The contiguous group to the southwest, No. 12, shows the lowest residuals of any group, the separate solutions are sharply differentiated and the depth is the greatest in the United States. On the side of this area, the gravity anomaly at St. Paul, 0.059 dyne per gram, is, next to Seattle, the largest found thus far in the United States. It may be concluded, then, that in this part of the continent, undisturbed by igneous activity or mountain-building since the pre-Cambrian, the depth of the zone of rigidity appears to be very great. The irregularities in residuals in group 5 may date from the Keweenawan period, when enormous masses of basic and therefore heavy magmas were intruded and extruded in the Lake

Superior region. If such be the case it shows the long endurance of strains borne by this part of the earth. In the almost universal epeirogenic movements which marked the close of the Tertiary and opening of the Pleistocene, the Lake Superior basin showed notable downwarping, its bottom being now beneath the level of the sea. It formed a trough which directed the flow of glacial ice. The latter must have scoured it clean but can hardly be ascribed as the cause of the existence of the basin. The crust movements have doubtless been in the direction of relief of stress, but the relief has been but partial; geodetic investigation reveals that the age-long load is yet borne.

DEPARTURES FROM ISOSTASY SUSTAINED BY RIGIDITY IN THE ZONE OF COMPENSATION

It was concluded under the last topic that the rigidity over certain parts of the earth probably carries the zone of possible compensation as deep as 300 km. even under the assumption of uniform rate, an assumption which tends to minimize the depth; whereas in other regions under that hypothesis it is less than 100 km. in depth. This raises the question whether the regional departures from isostasy are carried as strains within the zone of compensation or are transferred in part to the deeper body of the earth. There are reasons for believing that the former is the case, pointing by inference to a zone of markedly diminished rigidity between the rigid lithosphere and still more rigid centrosphere.

The geodetic evidence consists in the large values of the squares of the residuals for solution B, the solution which postulates extreme rigidity and compensation at infinite depth. For the whole United States, as shown in the Table XIX, p. 293, the mean value of the squares of the residuals for solution B is 10.7 times the value for solution H. But for group 12, that for which the most probable depth of compensation is 305 km., the distinction is still greater; solution B showing a mean-square residual 28 times greater than for solution E. Dividing in this way the value for solution B by the value for the most probable solution, and taking the mean for all those groups which indicate a depth of compensation greater than the average for the United States, it is found that the ratio is twice

as great for the groups with deep compensation as for the United States as a whole. That is, the groups with deep compensation, instead of showing a leaning toward solution B show on the contrary more definitely that it is not true. The hypothesis of uniform compensation complete at a certain depth appears to be more nearly true for regions with deep compensation than for shallow compensation. This does not mean, however, a lesser rigidity of the crust for the regions with deep compensation, their high capacity to carry strain being shown by the large gravity anomalies which are found in places within them.

There seems to be no evidence, however, that the zone of diminished rigidity is sharply bounded or is marked by real liquidity. It is doubtless due to the gradual rise of temperature with depth, overcoming within a certain zone the influence of the increasing pressure. Seismologic and tidal evidences show, furthermore, that under stresses of relatively brief duration the earth acts as a unit and as an elastic rigid body. The physical condition of the zone of low rigidity may approach that of a highly viscous fluid, the time element thus entering within these limits as a fundamental factor. This zone is incapable of bearing pronounced strains for long periods in the manner of the zone above. In geologic operations it thus serves to separate the mode of expression of forces generated below from those originating above this level. The former give rise to the great compressive movements in the outer zone, the latter to the vertical movements not determined by tangential compression.

INTERPRETATION OF DEFLECTION RESIDUALS IN TERMS OF MASSES

On p. 59, paper of 1909, Hayford shows that the actual deflections of the vertical average only one-tenth of what they would be if the continent and the portions of the ocean basins which were included in the calculations were both underlain by matter of the same density and the relief sustained wholly by the rigidity of the crust. The effect of the topography calculated on this assumption —that the density is uniform and the larger as well as the smaller features are sustained by rigidity—gives what is known as the topographic deflections. These, as stated above, average ten times

the value of the actually observed deflections. The surface may be regarded, therefore, as nine-tenths compensated by variations of density. The details for the five more significant groups are given below:[1]

TABLE XX

1	2	3	4	5	6	7	8
No.	Area of Group	No. of Stations in the Group	Probable Depth of Uniform Compensation in Kilometers	Mean of Topographic Deflections without Regard to Sign	Mean Residual of Solution H without Regard to Sign	Value in Sixth Column Divided by Value in Fifth Column	Percentage of Completeness of Isostatic Compensation for Solution H
12....	Parts of Minn., N.Dak., S.Dak., Neb., Kan.......	36	305	8.23	2.17	0.26	74
8....	Parts of Utah, Nev., Cal....	42	66	32.23	3.57	.11	89
10....	Cal., southern part........	57	126	65.44	3.91	.06	94
9....	Cal., northern part........	60	176	60.50	2.93	.05	95
14....	Northern Cal., western Ore., and Wash.............	37	84	53.68	3.37	.06	94
	Whole United States......	733	122	30.37	2.91	0.10	90

Group 12 gives the greatest depth for uniform compensation. By using the residual for Solution E, 2.09, the percentage of completeness of compensation would have been 75, a trifle more than for Solution H, but still next to the least perfect in the United States.

Group 8, the Great Basin region, has the lowest depth of compensation but shows about the average approximation to isostatic equilibrium.

Groups 10, 9, 14 comprise the Pacific Coast Ranges. They give the highest topographic deflections of the United States, doubtless on account of the great relief of the ocean basin and continental border, but the actually observed deflections do not differ greatly from group 8 or the mean for the whole United States. The result is that in this mountain region bordering the continent the degree

[1] Taken from pp. 56, 58, 69, and illustration No. 2, Hayford, 1909.

of completeness of compensation is the highest in the United States.

On the basis of the figures for the whole United States Hayford writes: "The average elevation above mean sea-level being about 2,500 feet, this average departure of less than one-tenth from complete compensation corresponds to excesses or deficiencies of mass represented by a stratum only 250 feet (76 meters) thick on an average."[1] It is this last statement, interpreting the deflection in terms of mass, which has meaning to the geologist. It has been widely quoted as perhaps the chief geologic result of the work and yet the writer believes that it is without basis. By an oversight of the author he misinterprets his results. If the present writer is correct in making this statement it should not be taken, however, as a criticism of the mathematical portion of the work.

The sea-level is from the standpoint of the problem of isostatic compensation but little more than a datum surface. Imagine the ocean water to be converted into rock of density 2.7 of the same mass as the water and resting on the present ocean bottom. Every thousand feet of water would be replaced by 380 ft. of rock. Then the sea-level surface after this transmutation is seen to lose all real significance[2] To show the fallacy of taking this level as a basis for interpreting the departures from compensation in terms of thicknesses, let attention be given to groups 1, 2, 3, 4, 6, 11,[3] which cover the United States east of the Mississippi River. The average departure of these from compensation is 0.11, which on the basis of Hayford's statement means that the surface on the average departs but 275 ft. from the level which would give complete isostatic equilibrium on the hypothesis of uniform distribution of compensation to a depth of 122 km. If, however, this eastern third of the United States be regarded by itself, its average elevation may be assumed as 1,000 ft. (it is probably less). By the same reasoning as Hayford applied to the whole United States, 11 per cent of this is 110 ft. Therefore although the average deflections are slightly

[1] 1909, p. 59.

[2] More accurately, the equivalent rock should be imagined as suspended at the mean depth of the water, but the effect of the difference in level is negligible upon the topographic deflection.

[3] 1909, p. 59.

greater than for the United States as a whole, it would be concluded that for the region east of the Mississippi the departure from the levels giving complete compensation averages not more than 110 ft. instead of the 275 ft. previously stated.

Or, again, imagine a rise of ocean-level so that the average elevation of this part of the continent is reduced to 100 ft. without changing the detail of the topography. The deflections would suffer only small alterations due to the added mass of water. Although the crust remained without change, the same reasoning would then lead to the conclusion that the topography departed on the average but 11 ft. from the levels which would give complete compensation.

In computing the influence of the topographic irregularities and their compensation upon the deflection of the vertical, all the topography was taken into account up to a radial distance of 4,125 km. from each station. This radius is approximately the length of 37° of latitude. It embraces the Pacific Ocean out to the Hawaiian Islands and to ten degrees south of the equator, and the Atlantic Ocean out to the Azores. The relief within this region ranges from −8,340 m. north of Porto Rico to +6,220 m. in Mount McKinley, +6,247 in Chimborazo; a total differential relief of about 14,590 m. About one-half of the topography surrounding the coast stations consists of ocean bed. Even for the stations in Minnesota, farthest removed from the sea, about one-third of the surrounding topography within the limits is deep ocean, but lying at a greater distance and carrying lesser influence. The average depth of the oceans influencing the deflection of the station at mean distance inland may be assumed for purposes of illustration to be about 5,000 meters. This depth of water is equivalent in mass to 1,900 m. of rock of density 2.67, leaving an effective ocean depth of 3,100 m. Add the mean continental elevation of 760 m. to this, and 3,800 to 3,900 m. represents about the effective mean relief between continent and ocean. On coast stations this differential relief has greatest influence. For inland stations the several portions of the continent have proportionately more effect. For the United States as a whole it is this relief of between 3,500 and 4,000 m. between continent and ocean, more than the relief between the major features of the continent, which is nine-tenths compen-

sated by the corresponding variations in crustal density, not the 760 m. which is the average elevation of the United States above sea-level.

It is the belt of Pacific coast stations which measures more closely than other groups the degree of compensation accompanying the continental relief above the ocean bottoms. These stations lie in groups 10, 9, and 14, for which the mean residuals are but 0.06, 0.05, and 0.06 of the mean topographic deflections respectively. These residual deflections indicate that for this coastal zone the departures from complete compensation amount to but 5 or 6 per cent. If the mean effective relief which controls this be assumed as 4,000 m., then the mean departure from equilibrium is represented by a mass 200 to 240 m. thick, approximately between 650 and 800 ft. On the other hand, groups 5 and 12 are those farthest removed from the ocean basins and their deflections are controlled most largely by the internal continental relations. For them the departures from complete isostatic compensation as measured by the ratio of the mean residuals to the computed topographic deflections amount to 42 and 26 per cent. The mass to which this is equivalent may be no greater than the 5 per cent departure on the Pacific coast. These estimates fall into the same order of magnitude as that of the masses represented by the gravity anomalies.

This reconnaissance of the problem is sufficient for present purposes. It is readily seen that even greater difficulties stand in the way of a precise statement regarding the equivalence of mass corresponding to deflections of the vertical than arose in the interpretation of the gravity anomalies. The residual for each observed deflection is the sum of the influences of all the excesses and deficiencies of mass as compared to solution H on all sides of a station. The effect of each unit varies inversely with the square of the distance and directly with the sine of the angle which the line of force makes with the horizontal passing through each station. A combination of the data from the measurements of the intensity of gravity with those of the deflections of the vertical would apparently be necessary to state for each region the equivalence in terms of mass which is implied by the residual at each station.

MAXIMUM LOADS INDICATED BY ANOMALIES

Hayford and Bowie consider that 0.0030 dyne of anomaly may be regarded as equivalent to 100 ft. of rock possessing a density of 2.67. From the previous considerations it would seem that this is probably too high for a mean figure, but may apply to certain areas, especially those with extremely broad boundaries. In other regions 0.003 may be far too high, since it is shown under the topic "Variable or Constant Depth of Compensation" that in certain parts of the United States the depth of the zone of compensation probably goes notably deeper than in other parts and the density may be distributed either nearly uniformly or with considerable irregularity. The greatest depth of compensation indicated for any region is 305 km. A unit thickness of mass uniformly distributed to this depth and to a radius of 166.7 km. would give but 0.0014 of anomaly instead of 0.0024 as given by a depth of 114 km., or 0.0030 as taken by Hayford and Bowie. For general use 0.0024 dyne is perhaps the best value, corresponding to a uniform distribution of a unit excess or defect of mass to a depth of 114 km. and to a radial distance of 166.7 km. For the mean anomaly of 0.018 this would give 750 ft. of elevation as the mean departure of the surface of the United States above or below the position giving isostatic equilibrium, instead of 600, or more exactly, 630 ft. as taken by Bowie. The largest known anomaly in the United States is at Seattle, -0.093. This corresponds to a defect in mass equivalent to a stratum 4,000 ft. thick if the divisor is 0.0024, a stratum 3,200 ft. thick if the divisor is 0.0030. At Olympia, but 50 miles or 80 km. distant, the anomaly is $+0.033$, corresponding to excesses of mass of 1,375 or 1,100 ft., according to the divisor. The difference of regional load between Olympia and Seattle becomes 5,375 or 4,300 ft.

But these relations of unit thickness of mass to the gravity anomaly are based on the assumption that the excess or deficiency of mass extends to as great a radial distance as 166.7 km. radius. This minimizes the thicknesses or densities needed to account for the anomalies above what would be required for a more local concentration of mass. But an inspection of the distribution of gravity and deflection residuals shows that in many cases the masses

producing the greater disturbances have much smaller size. This is especially striking in the case of the largest negative anomaly in the United States, that at Seattle, only 50 miles from the large positive anomaly at Olympia. The latter is surrounded on all sides by negative anomalies as follows:

DISTANCES FROM OLYMPIA, WASHINGTON

Astoria, Ore.	76	miles	S.W.	−.013 dyne anomaly
Heppner, Ore.	195	"	S.E.	−.027 " "
Skyhomish, Wash. .	84	"	N.E.	−.028 " "
Seattle, Wash.	50	"	N.N.E.	−.093 " "

The excess of mass which exists in the vicinity of Olympia, above that required for compensation under solution G, must therefore be much less than 166.7 km. (102.5 miles) in radius. The same is doubtless true of that excessive deficiency which exists at Seattle, since the anomaly sinks to less than one-third the value at Skyhomish only 45 miles east, and changes to a large positive anomaly at Olympia, 50 miles south-southwest.

The large positive mass in the vicinity of Olympia must diminish appreciably the effect of the still larger negative mass in the vicinity of Seattle. The latter with the other surrounding negative masses must diminish still more the anomaly due to the positive mass at Olympia. Furthermore, it is highly improbable that the observations at Seattle should happen to be made at the point of really maximum anomaly. Let the very moderate assumption be made then that the abnormal Seattle mass as a unit by itself would give a maximum anomaly of −0.100 dyne. It would doubtless give more. Let limiting assumptions be made as to the dimensions and density of this mass such that the actual volume and density are quite probably embraced somewhere within these limits. Tables XXI[1] and XXII show the results of such assump-

[1] Table XXI is readily derived from Table X, Part III. Take, for example, the cylinder of radius 1,280 meters, depth of 1,000 feet, and density 0.267. Multiply its dimensions by 30 and the volume of each unit portion will be increased by the cube of 30. The attraction of each unit of mass on the given point will vary inversely with the square of the distance and will therefore be diminished by the square of 30. The anomaly will consequently increase directly with the dimensions, provided that the density remains constant. This gives the basis for the calculations in column 2, Table XXI.

tions. In Table XXI the attracting mass is supposed to have the form of a vertical cylinder. With a given anomaly the deficiency

TABLE XXI

VERTICAL CYLINDERS GIVING A NEGATIVE GRAVITY ANOMALY OF 0.100 DYNE AT CENTER OF TOP SURFACE OF CYLINDER

1	2	3	4	5
Diameter	76.8 km.	51.2 km.	102.4 km.	51.2 km.
Depth	9.15 km.	30.5 km.	61.0 km.	61.0 km.
Density	− 0.31	− 0.15	− 0.07	− 0.12
2.80−Density	2.49	2.65	2.73	2.68
Thickness of cylinder of same area and mass, but density 2.67	1,080 m. 3,550 ft.	1,700 m. 5,600 ft.	1,700 m. 5,600 ft.	2,770 m. 9,080 ft.
Anomaly per 100 feet of mass of density 2.67 expanded to depth of cylinder as given in second line	0.0028 dyne	0.0018 dyne	0.0018 dyne	0.0011 dyne

TABLE XXII

SPHERES GIVING A NEGATIVE GRAVITY ANOMALY OF 0.100 DYNE AT POINT VERTICALLY ABOVE ON THE SURFACE OF THE EARTH

1	2	3	4	5
Diameter	50. km.	100. km.	50. km.	100. km.
Depth to center	25. km.	50. km.	32. km.	64. km.
Density	− 0.144	− 0.072	− 0.236	− 0.118
2.80−Density	2.66	2.73	2.56	2.68
Length of polar axis of oblate spheroid of same equatorial section and same mass, but density 2.67	2,700 m. 8,850 ft.	2,700 m. 8,850 ft.	4,420 m. 14,500 ft.	4,420 m. 14,500 ft.
Anomaly per 100 feet of polar axis of mass at density 2.67 if expanded to diameter of sphere	0.0011 dyne	0.0011 dyne	0.0007 dyne	0.0007 dyne

of mass will be least if the cylinder extends from the station downward instead of being at a greater depth. Furthermore, for a given volume and density of cylinder the gravitative force will vary according to the ratio of the depth to the diameter.

Let H = depth
Let $2R$ = diameter
Let F = gravitative force

Then $\pi R^2 H=$ the volume, a constant. To find the ratios of H to R which give maximum attraction for unit mass

Let $R^2 H=1$ and solve for various values of R the equation

$$F=2\pi\rho\gamma\left[\frac{(R^3+1)-\sqrt{R^6+1}}{R^2}\right].$$

Several solutions are as follows:

For $H=0.75$, $R=1.15$; $F=(2\pi\rho\gamma)\ 0.523$
$H=1.00$, $R=1.00$; $F=(2\pi\rho\gamma)\ 0.586$
$H=1.50$, $R=0.81$; $F=(2\pi\rho\gamma)\ 0.609$
$H=2.00$, $R=0.71$; $F=(2\pi\rho\gamma)\ 0.586$
$H=4.00$, $R=0.50$; $F=(2\pi\rho\gamma)\ 0.200$

This shows that the gravitative force is a maximum for a cylinder of constant volume and mass in which the depth varies from one-half the diameter to four-thirds the diameter. The force varies but slightly between those limits. The cylinders of columns 3, 5, and 6, of Table XXI, lie within these limits. Thus all the assumptions thus far made favor the minimization of the negative load which produces the Seattle anomaly.

Taking the mean density of the outer part of the lithosphere as 2.80 it is seen that the cylinder of column 2, Table XXI, has a density below that of the lightest rock-making minerals and would require the existence of a molten magma or of abnormal pore space to great depth. It may therefore be eliminated as not probable. Cylinders 3, 4, and 5 show densities within the limits of granite, the lightest of the abyssal igneous rocks. It may be concluded, therefore, that the deficiency of mass, if of approximately cylindrical form, is equivalent to a negative load of between 5,000 and 10,000 feet of rock, extending over a distance of from 50 to 100 km., or a somewhat less local load superposed upon a broad but small regional load of the same sign. The nature of the assumptions has been such that we may conclude with confidence that the Seattle anomaly corresponds to at least 5,000 feet of rock and may reach a considerably higher figure, perhaps 10,000 feet. Furthermore, Hayford's unit mass, extending to the areal limits, 100 feet thick and density 2.67, would here produce an anomaly as low as between 0.0010 and 0.0020 dyne.

Instead of a cylinder suppose the mass which produces the deficiency of gravity to approximate more to the form of a sphere. The results are shown in Table XXII. In columns 2 and 3 the sphere is tangent to the surface, a position diminishing the mass for a given anomaly. In columns 3 and 4 the top of the sphere is 7 and 14 km. deep respectively. The low density of column 4 shows it to be beyond the limiting conditions. The load, though negative in sign, is seen to be equivalent in order of magnitude to the greater volcanic piles; 30 to 60 miles in diameter, 9,000 to 14,000 feet in height for rock of density 2.67. The anomaly produced by the unit mass of 100 feet thickness and density 2.67, considered here as 100 feet of polar diameter for a spheroid of the given horizontal dimensions, ranges between the low values of 0.0007 and 0.0011 dyne.

From a consideration of these two tables it is seen that the large anomalies require either a variation of mass equivalent to as much as 5,000 feet of rock extending over some thousands of square miles or to 10,000 feet of rock, more or less, extending over 1,000 square miles, more or less. These tables determine the order of magnitude, but the data are not sufficient to permit a more accurate solution of the problem.

Thus this detailed examination of the anomalies in the region of Seattle shows that the divisor of 0.0030, as taken by Hayford and Bowie, or 0.0024, as considered here the best for general use, is too high for the more limited areas of high anomaly. The latter may be regarded as made up in part of a regional portion for which the divisor of 0.0024 would be applicable and a local portion for which the divisor is probably not over 0.0015. As a mean value, for the more limited areas of large anomaly the amount due to the unit thickness of 100 feet of rock of density 2.67 should apparently not be taken as over 0.0020 dyne.

In forming conceptions as to the uncompensated vertical stresses existing widely in the earth's crust it is important to know the maximum range of departures from the mean stress as well as the latter. These can be studied well in Fig. 5.[1] The mean of fourteen maxima of defect of gravity is -0.033, the mean for eleven

[1] Fig. 5, p. 153, Part II; also see Hayford and Bowie, pp. 107–8.

areas of excess of gravity is +0.034. With the exception of the Seattle stations with an anomaly of −0.093, none reach a value of 0.060. It is thus seen that the average notable maximum is not far from twice the mean anomaly. Even by using a uniform divisor of 0.0024 or 0.0030 to convert anomalies, regional departures of load amounting to 1,300 or 1,500 ft. over areas of several square degrees are found to be not uncommon. Over smaller areas the loads rise to about three times the mean, and at Seattle to five times the mean. These figures of course do not measure simply the elevations or depressions of uncompensated erosion features. On the contrary, if the hills and valleys be imagined as smoothed out, then the resulting mean surface would be out of isostatic equilibrium in the same direction over distances amounting frequently to hundreds of kilometers and attaining maximum departures too low or too high over smaller areas by these figures.

But an inspection of the contour map of gravity anomalies (Fig. 5) shows that the large anomalies, those of 0.040 dyne or above, are all located by Hayford and Bowie as centers of maximum anomaly, though the nearest adjacent stations average as much as 100 miles distant. Between the widely spaced stations, the anomaly gradients are gentle. But where the stations form a series closer together, as that from the city of Washington to New York City, the gradients are seen to be steeper and more irregular. It is to be presumed that a further multiplication of stations would show increased complexity over the whole country and reveal maxima higher than those now recorded. The value of the mean anomaly without regard to sign should furthermore increase somewhat through the discovery of additional areas of maximum value. Areas of regional positive or negative anomaly would persist in something of their present size, but within broad areas of anomaly of one sign should be discovered smaller areas of opposite sign which are now unknown. Upon the completion of such a detailed survey the high anomaly of Seattle would not appear so exceptional as it does at present.

The chart of the residuals of Solution H[1] shows within the larger areas of like deflection of the vertical many large and sharp

[1] Illustration No. 3, Hayford, Supplementary Paper, Bowie, Illustration No. 5.

variations in value and in direction. The resultants of the plotted arrows point toward the centers of exceptional mass and their rapid changes in value and direction point toward the existence of many comparatively shallow masses. The epicentral points above such masses are those where the gravity anomaly, Fv, is at a maximum. If a hidden mass may be regarded as approaching a spherical form and has its center at depth D, the following relations exist between the value of the gravity anomaly and the distance x from the epicenter:

$$
\begin{array}{lll}
Fv=\text{Maximum} & \text{for} & x=0.00\,D \\
Fv=.75 \text{ max.} & `` & x=0.46\,D \\
Fv=.50 \text{ max.} & `` & x=0.77\,D \\
Fv=.25 \text{ max.} & `` & x=1.23\,D
\end{array}
$$

If, for example, an approximately spherical mass has its center at a depth of 32 km., .005 of the earth's radius, the anomaly Fv will fall to half-value at a distance of 25 km. from the epicenter. If the center is 64 km. deep, the anomaly will fall to half-value at 50 km. from the epicenter. Between stations located 100 km. apart by far the greater number of real maxima would be missed, and in so far as they depended upon masses in the upper half of the zone of compensation the indicated maxima would at most places be less than one-half the real maxima.

The stresses acting within the crust owing to excesses or deficiencies of mass are not so concentrated and therefore not quite so great as if those abnormalities of mass existed as surface loads of rock of density 2.67 in the manner imagined for the interpretation of anomalies.[1] Nevertheless to gain a conception of the meaning of the gravity anomalies, imagine the present compensated topography to be smoothed out to sea-level and the variations of mass away from isostatic equilibrium to become variations of volume upon its surface. The anomaly contours will then become topography contours, the line of zero anomaly will become the datum plane. The values in mass to be assigned to the successive anomaly contours can only be given in mean figures. It has been shown however in Part III that balanced vertical irregularities of density

[1] The relations of mass and its distribution to the resulting stresses will be considered in a later part.

do not play a large part in causing gravity anomalies. It will be shown later that neither can nucleal heterogeneity below the 200 to 300 km. level of isostatic compensation account for a large part. The anomalies represent in greater part real departures from isostasy and, as shown in this section, the limited areas of high anomaly are to be interpreted as implying on the whole a local load in higher ratio to anomaly than do the broad areas of anomaly. The average relation thought to exist is shown then in the following table:

TABLE XXIII

AN ESTIMATED AVERAGE RELATION OF ANOMALY CONTOURS TO CONTOURS OF EQUIVALENT ROCK MASSES OF DENSITY 2.67

Anomaly Contour, Positive or Negative	Assumed Divisor for 100 Feet of Rock upon a Level Surface	Equivalent Contour in Feet, Positive or Negative
.020	.0025	800
.040	.0023	1700
.060	.0020	3000
.080	.0018	4500
.100	.0016	6300

Upon conversion of a detailed anomaly map, if such existed, into the equivalent topographic contour map by means of such ratios as those given in Table XXIII, the whole United States with its compensated topography previously smoothed out to sea-level would be reconverted into a roughly mountainous country with no notable distinction between what are now the central plains and mountainous border regions of the continent. On to broad plateaus or basins upward of 1,000 feet from the mean elevation would be added higher elevations and depressions. The extreme differential relief would probably be in the neighborhood of two miles though the average departure without respect to sign from the mean surface of the geoid would probably be between 800 and 1,000 feet. Though everywhere as irregular as a mountainous country, there would be little or no relief of the mean level of this hypothetical surface above the ocean bottoms and no such broad and high masses as the Cordilleran plateaus would remain within

its limits. The major relief of the continent above the ocean bottoms would be about nine-tenths eliminated and the mean elevation of all areas as great as several hundred miles in width would be reduced to a small figure. This is the effect of isostasy. But within these unit areas which measure the limits of regional compensation would everywhere rise a rolling mountainous surface.

Imagine the hypothetical surface as broad as the United States, concealed from view by an impenetrable envelope of cloud, and aerial explorers to sink a sounding line to this invisible land at 124 places chosen at random. The resulting contour map compiled from these soundings would yield a much smoothed and flattened surface such as is shown in the contour map of gravity anomalies. Many of the soundings taken really on mountain slopes, because they were the highest of those made, might be casually interpreted as located on mountain peaks. The latter, standing sharp and high, would be missed save for an occassional lucky chance of the sounding line.

Interpreted in terms of weights and stresses, it is seen that even the parts of the continent appearing to the eye as plains long in geologic quietude really conceal within them strains as great as those imposed by the weight of mountains. That these great strains have been born for geologic ages, in many localities probably from the Archeozoic, gives a surprising conception of an enduring rigidity and elasticity of the crust wholly at variance with certain current doctrines regarding the weakness of this zone. It is not here found to be a failing structure.

On p. 81 Hayford and Bowie give the new-method anomalies for sixteen stations not in the United States. An abstract is given below of the greater anomalies from that table with the addition of Seattle. The thickness of stratum taken as corresponding to the anomaly is also added. This thickness, if the compensation is uniform with depth, measures the distance by which the earth's surface is out of isostatic equilibrium at those points. A plus sign indicates an excess of mass and a consequent tendency to sink, resisted by rigidity; a minus sign a defect of mass and therefore the existence of an upward strain.

The divisor 0.003 dyne of anomaly, taken as the equivalent

of 100 feet of abnormal mass of density 2.67, is Hayford's figure. As previously discussed it is thought to minimize too much the thickness of equivalent rock. It is given, however, for comparison with the column derived from the use of 0.0024 as a divisor. This is regarded as a better average figure, but for some cases at least, as shown for Seattle, this also may give too low a result.

TABLE XXIV

Number and Name of Station	Elevation in Meters	New-Method Anomaly	Thickness of Equivalent Stratum on Land in Feet	
			0.003 Anomaly =100 feet	0.0024 Anomaly =100 feet
2. Tonga plateau, Hecker, at sea..............	−2,700	+0.255	+8,500	+10,625
4. Tonga deep, Hecker, at sea.................	−6,500	− .184	−6,130	− 7,660
9. Mauna Kea, Hawaiian Islands.............	+3,981	+ .183	+6,130	+ 7,660
10. Hachinohe, Japan......	+ 21	+ .110	+3,670	+ 4,590
13. Sorvaagen, Norway....	+ 19	+ .146	+4,870	+ 6,090
Seattle, United States..	....	−0.095	−3,170	− 3,960

It is seen that the excesses of mass indicated for Mauna Kea and at Sorvaagen are each comparable in equivalent thickness and extent to the maximum thickness of the Niger Delta if measured by rock upon land, 5,450 feet. The departures from equilibrium at Hachinohe, Japan, and Seattle, United States, are comparable in thickness and area to the burden of the Nile Delta, the weight in air of 3,600 to 4,200 ft. of rock. In weight as in area, therefore, these deltas are seen to impose burdens on the crust no greater than are found, by means of geodetic observations, to exist in certain other regions where geologic evidence had not revealed them. The accuracy of Hecker's method for determining the intensity of gravity at sea has been called into question by Bauer[1] so that, until

[1] "On Gravity Determinations at Sea," *Amer. Jour. Science* (4), XXXI (1911), 1–18. "Hecker's Remarks on Ocean Gravity," *Amer. Jour. Science* (4), XXXIII (1912), 245, 248.

this question is settled by geodesists, equal weight should perhaps not be attached to the figures given for the departures from isostasy shown over the Tonga plateau and Tonga deep. Neither is the area of these departures known, though the areas of the plateau and deep are large. These regions are seen, however, to indicate considerably higher departures from isostasy than the measurements determined from the deltas of the Nile and Niger. The latter, therefore, perhaps do not measure the full strength of the crust.

Major H. L. Crosthwait has applied Hayford's methods to the investigation of isostasy in India.[1] The residuals of the deflections of the vertical serve as a measure of the degree of compensation existing in the United States as compared in India and are as follows:

UNITED STATES OF AMERICA

Group S.E., mean residual	− 0″74
Group N.E., mean residual	− 1.04
Group Central, mean residual	− 1.66
Group W., mean residual	− 4.02

INDIA

Region No. 1, Himalaya Mountains, mean residual	−16″
Region No. 2, Plains at foot of Himalaya Mts., mean residual	− 2.
Region No. 3, N.E., mean residual	+ 8.
Region No. 4, Central, mean residual	+ 5.
Region No. 5, N.W., mean residual	+ 4.
Region No. 7, W., mean residual	− 3.
Region No. 8, E., mean residual	− 2.
Region No. 9, S., mean residual	+ 1.

It is seen that the residuals average several times as great in India as in the United States, which leads him to conclude that "Speaking generally it would appear that isostatic conditions are much more nearly realized in America than in India, i.e., if we are to take the smallness of the residuals as an indication of the completeness of isostatic compensation."[2] Colonel Burrard, utilizing the Hayfordian computations, points out the existence of zones

[1] *Professional Paper No. 13, Survey of India*, 1912.

[2] *Op. cit.*, p. 4.

in India where the deflections of the plumb-line are actually in opposition to the directions called for by isostasy.[1] The major elements of the relief, the Himalayas, the plateau of India, and the surrounding ocean basins are of course largely compensated, but these figures show that in detail the hypothesis of complete isostasy is very far from the truth. Crosthwait suggests that the explanation for the difference between the United States and India probably lies in the magnitude of the recent upheavals of the crust in that part of the globe. Nevertheless such upheavals cannot exceed the strength of the crust, and in India, therefore, perhaps may be better observed than in the United States the maximum strains which the earth is competent to endure.

It may be concluded, therefore, that the convergence of geodetic evidence shows the crust to be competent to sustain loads measured by the weight of several thousand feet of rock extending over circular areas some tens of thousands of square miles in area. This is a measure of crustal strength twenty, fifty, or even a hundred fold greater than that advanced in recent years by the leading champions of high isostasy.

FURTHER GEODETIC WORK NEEDED FOR GEOLOGIC PROBLEMS

It has been the intention in the preceding analysis to show two things: first, that the data set forth by Hayford and Bowie are of great value to geology and establish new methods of research, but, second, that the difficulties inherent in the observations and their mathematical treatment, and the fewness of the stations in comparison with the heterogeneity of the earth, are such that the conclusions from the geologic study of deltas in the first part of this paper are as convincing and perhaps as accurate as the present results of the geodetic studies. The latter, however, opens for the whole earth a field of investigation which the geologic evidence covers very locally and imperfectly, a world-wide field which should be pursued for the geologic as much as for the geodetic bearings.

By means of the divining rods of pendulum and plumb-line the heterogeneities of mass and the loci of strain in the outer crust of

[1] "On the Origin of the Himalaya Mountains," *Professional Paper No. 12, Survey of India*, 1912.

the earth should be sought out and measured in detail. For this work it would seem that many new stations would have to be established; in groups so as to reduce the errors of each locality; in sets so as to attack particular phases of the problems. For example, it would appear that gravity stations should be located in pairs close together and of as great a difference in elevation as possible. Certain stations should be located within areas of plateaus spared by circumdenudation, such as the Cumberland and Allegheny plateaus; others should be located in the broad erosion basins. Deflection stations should be located on the lines separating regions of erosion from those of circumdenudation, and also on the lines separating areas of upwarp from those of downwarp. A network should inclose, finally, all centers of marked gravity anomaly or topographic deflection. Such an increase in the number of stations would permit the introduction of simple hypotheses of variable depth and rate and regional limits of compensation. But such an extensive program is within the reach only of some reseach institution. It needs the co-operation of geologists and geodesists. The location of stations with respect to surface features and their geologic history should be controlled by the geologist. The density of the rocks to the limits exposed by the structure should also be determined by him. The geodesist, on the other hand, should seek out the hidden heterogeneities in the crust and guide the details of the work.

[*To be continued*]